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SOME COMMON FIXED POINT THEOREMS IN INTUTIONISTIC FUZZY 2-METRIC SPACES UNDER STRICT CONTRACTIVE CONDITIONS

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Abstract

The aim of this paper is to prove the existence and uniqueness of common fixed point theorem in intuitionistic fuzzy2 metric space under the contractive condition .In this paper we modify and extend the results of Sharma and Bamboria [23].

Keywords: Fixed point, Metric Space , Fuzzy Metric space, Intutionistic Fuzzy metric space,Intutionistic fuzzy 2 metric space, Property S-B , t-norm, t-conorm.

AMS Mathematics Subject Classification (2010): 54H25, 47H10

1. INTRODUCTION

The concept of fuzzy sets was introduced initially by Zadeh [26] in 1965. Since then, to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and its applications. In 1975, Karmosil and Michalek [16] introduced the concept of a fuzzy metric space based on fuzzy sets, Especially, Deng [8], Erceg [9], kaleva and Seikkala [15], Kramosil and Michalek [16] have introduced the concept of fuzzy metric spaces in different ways. This notion was further modified by George and Veermani [11] with the help of t-norms. Many authors made use of the definition of a fuzzy metric space in proving fixed point theorems. In 1976, Jungck [13] established common fixed point theorems for commuting maps generalizing the Banach's fixed point theorem. Sessa [21] defined a generalization of commutativity, which is called weak commutativity. Further Jungck [14] introduced more generalized commutativity, so called compatibility. Mishra et. al. [17] introduced the concept of compatibility in fuzzy metric spaces.

Atanassov [1-5] introduced the notion of Intuitionistic fuzzy sets and developed its theory. Park [19] using the idea of intuitionistic fuzzy sets to define the notion of intuionistic fuzzy metric spaces with the help of continuous t-norm and continuous t-conorm as a generalization of fuzzy metric space. Gahler [10] introduced and studied the concept of 2-metric spaces in a series of his papers. Iseki et. al. [13] investigated, for the first time, contraction type mappings in 2-metric spaces. In 2002 Sharma [18] introduced the concept of fuzzy 2-metric spaces. Mursaleen et. al. [18] introduced the concept of intuitionistic fuzzy 2-metric space. Sharma, Sharma and Iseki [25] studied for the first time contraction type mappings in 2-metric spaces.

The aim of this paper is to define a new property that generalize the concept of non-compatible mappings and give some common fixed point theorems in Intutionistic fuzzy 2-metric space under strict contractive conditions. We extend results of Sharma and Bamboria [23].

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2. Preliminaries

Definition 2.1 [24]. A binary operation $* :[0,1]\times[0,1]\times[0,1] \rightarrow [0,1]$ is called a continuous t-norm if ([0,1],*) is an abelian topological monoid with unit 1 such that $a_1*b_1*c_1 \le a_2*b_2*c_2$ whenever $a_1 \le a_2$, $b_1 \le b_2$, $c_1 \le c_2$ for all a_1, a_2, b_1, b_2 and c_1, c_2 are in [0,1].

Definition 2.2 [10]. Let X be a non-empty set. A real valued function d on X ×X × X is said to be a 2-metric on X if

(a) For given distinct elements x, y of X, there exists an element z of X such that d(x, y, z) = 0,
(b) d(x, y, z) = 0 when atleast two of x, y, z are equal,
(c) d(x, y, z) = d(x, z, y) = d(y, z, x) for all *x, y, z* in X,
(d) d(x, y, z) ≤ d(x, y,w) +d(x,w, z)+ d(w, y, z) for all x, y, z,w in X.
The pair (X, d) is then called a 2-metric space.

Example -2.1 : Let $X=R^3$ is a 2-metric such that d(x,y,z)= the area of a traiangle spanned by x,y,z, which may be given explicitly by the formula

$$d(x,y,z)=\left| x_1(y_2z_3-z_2y_3)-x_2(y_1z_3-y_3z_1)+x_3(y_1z_2-y_2z_1) \right|$$

where $x=(x_1, x_2, x_3)$, $y=(,y_1, ,y_2, ,y_3)$ and $z=(z_1, z_2, z_3)$

Definition 2.3[24]: The 3-tuple (X,M,*) is called a Fuzzy 2-metric space if X is an arbitrary set, * is a continuous *t*-norm and M is a fuzzy set in $X^3 \times [0,\infty)$ satisfying the following conditions : for all $x,y,z,u \in X$ and $t_1,t_2,t_3 > 0$.

(F2M-1) $M(x,y,z,0) = 0$,
(F2M-2) $M(x,y,z,t) = 1$, $t > 0$ and when at least two of the three points are equal,
(F2M-3) M(x,y,z,t) = M(x,z,y,t) = M(y,z,x,t), (Symmetry about three variables)
(F2M-4) $M(x,y,z,t_1 + t_2 +t_3) \ge M(x,y,u,t_1)* M(x,u,z,t_2)* M(u,y,z,t_3)$
(This corresponds to tetrahedron inequality in 2-metric space)

The function value M(x,y,z,t) may be interpreted as the probability that the area of triangle is less than *t*.
(F2M-5) $M(x,y,z, .):[0,1)\rightarrow [0,1]$ is left continuous.
(F2M-6) $\lim_{t\to\infty} M(x,y,a, t) = 1$ for all $x,y,a \in X$.

Example 2.2 [**24**] . Let (X,d) be a 2-metric space . Define a*b = ab (or a*b = min{a,b}) and for all $x,y \in X$ and t > 0,

$$M(x,y,a,t)=\frac{t}{t+d(x,y,a)} \qquad (1.a)$$

Then (X,M,*) is a fuzzy 2-metric space . We call this fuzzy metric M induced by the metric d the standard fuzzy metric .

Remark 2.1 . Since * is continuous, it follows from (FM-4) that the limit of the sequence in FM-space is uniquely determined.

Definition-2.4*:* A 5-tuple (X, M, *N*, *, ◇) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, ◇ is a continuous *t*-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$, s, t > 0,

(IFM-1) M(x, y, t) + N(x, y, t) ≤ 1
(IFM-2) M(x, y, t) > 0
(IFM-3) M(x, y, t) = 1 if and only if x = y
(IFM-4) M(x, y, t) = M(y, x, t)
(IFM-5) M(x, y, t) *M(y, z, s) ≤ M(x, z, t + s)

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(IFM-6) M(x, y, .) : $(0, \infty) \rightarrow (0, 1]$ is continuous
(IFM-7) N(x, y, t) > 0
(IFM-8) N(x, y, t) = 0 if and only if x = y
(IFM-9) N(x, y, t) = N(y, x, t)
(IFM-10) N(x, y, t) ◊N(y, z, s) ≥ N(x, z, t + s)
(IFM-11) N(x, y,.): $(0,\infty) \rightarrow (0, 1]$ is continuous
Then (M, N) is called an Intuitionistic fuzzy metric on X.
Note: M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non nearness between x and y with respect to 't' respectively.

Definition 2.5. A 5-tuple (X, M, N, *, ◊) is said to be an intuitionistic fuzzy2 metric space if X is an y non empty arbitrary set, * is a continuous t-norm, ◊ is a continuous t-conorm and M, N are fuzzy sets on $X^3 \times (0, \infty)$ satisfying the following conditions: for all x, y, z ,w ∈ X, r ,s, t > 0
(IF2M-1) M(x, y,z, t) + N(x, y,z, t) ≤ 1
(IFM-2) For given distinct elements x, y,z of X, there exists an element z of X such that M(x, y,z, t) > 0
(IF2M-3) M(x, y, z, t) = 1 if atleast two of *x,y,z* of X are equal *(i.e.* either *x=y* or *y=z* or *z=x)*
(IF2M-4) M(x, y, z, t) = M(*x,z, y, t*) = M(*y,z, x, t*)
(IF2M-5) M(x, y, z. r+s+t) ≥M (*x,y,w,r*)*M(*x,w, z, s*) * M(*w,y,z s*)
(IF2M-6) M(x, y,z .) : $(0, \infty) \rightarrow (0, 1]$ is continuous
(IF2M-7) N(x, y,z, t) < 0
(IF2M-8) N (x, y, z, t) = 0 if atleast two of *x,y,z* of X are equal *(i.e.* either *x=y* or *y=z* or *z=x)*
(IF2M-9) N(x, y, z, t) = N(*x,z, y, t*) = N(*y,z, x, t*)
(IF2M-10) N(x, y, z. r+s+t) ≥N (*x,y,w,r*)*N(*x,w, z, s*) * N(*w,y,z s*)
(IF2M-11) N(x, y,z, .): $(0,\infty) \rightarrow (0, 1]$ is continuous
Then (M, N) is called an Intuitionistic fuzzy2 metric on X and denoted by $(M,N)_{2.}$
Note: M(x, y,z, t) and N(x, y, z,t) denote the degree of nearness and the degree of non nearness between *x* and *y* with respect to '*t'* respectively.

Example : Let (X,d) is a 2- metric space. Denote a*b=ab and a◊b= min{1,a+b} for all a,b ∈[0,1] and M_d and N_d be fuzzy sets on $X^3 \times (0, \infty)$ defined by

$$M_d(x,y,z,t) = \frac{ht^n}{ht^n + m\,d(x,y,z,t)} \text{ and } N_d(x,y,z,t) = \frac{d(x,y,z,t)}{kt^n + m\,d(x,y,z,t)}$$

For all h,k, m,n ∈ R^+ . Then (X, M_d, N_d, *, ◊)is IF2M – space.

Definition2.6: Let (X, M, N, *, ◊) is an intuitionistic fuzzy2 metric space.
(a) A sequence $\{x_n\}$ in IF2M-space X is said to be **convergent** to a point
x ∈ X (denoted by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$) if for any k ∈ (0, 1) and t>0, there exist $n_0 \in N$ such that for all $n \geq n_0$ and a∈X, M(x_n, x,a,t)>1-k and N(x_n, x,a,t)< k .
That is $\lim_{n\to\infty}$ M(x_n, x,a,t)=1 and $\lim_{n\to\infty}$ N(x_n, x,a,t)=0 for all a∈X and t>0.
(b) A sequence $\{x_n\}$ in IF2M-space X is said to be Cauchy sequence if for any k ∈ (0, 1) and t>0, there exist $n_0 \in N$ such that for all m, $n \geq n_0$ and a∈X, M(x_m, x_n,a,t)>1-k and N(x_m, x_n,a,t)< k . That is $\lim_{n\to\infty}$ M(x_m, x_n ,a,t)=1 and $\lim_{n\to\infty}$ N(x_m, x_n,a,t)=0 for all a∈X and t>0.
(c) The IF2M-space X is said to be ***complete*** if and only if every Cauchy sequence is convergent.

Definition 2.7 :. A pair of mappings A and S is called weakly compatible in an Intutionistic fuzzy 2-metric space if they commute at coincidence points. ; i.e., if Tu = Su for some u ∈ X, then TSu = STu.

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Definition 2.8 : Let S and T be two self mappings of an Intuitionistic fuzzy2 metric space (X, M, N, *, ◊) . We say that S and T satisfy the property (S-B) if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

Example 2.2.: Let X = [0, +∞). Define S, T: X→ X by $Tx = \frac{x}{5}$ and $Sx = \frac{3x}{5}$, for all x in X. Consider the sequence $\{x_n\} = \{1/n\}$. Clearly $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = 0$. Then S and T satisfy the property (S-B).

Lemma 2.1[22] . For all $x,y \in X$, M(x,y,z, .) is nondecreasing and N(x,y,z, .) is non increasing.

Lemma 2.2[22]: If, for all $x,y,a \in X$, t > 0 and for a number $k \in (0,1)$,

$$M(x,y,a,kt) \geq M(x,y,a,t) \quad \text{and} \quad N(x,y,a,kt) \leq N(x,y,a,t)$$

then x = y.

Definition 2.9 : Let S and T be two self mappings of an Intuitionistic fuzzy metric space (X, M, N, *, ◊) . We say that S and T satisfy the property (S-B) if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

Example 2.3.: Let X = [0, +∞). Define S, T: X→ X by $Tx = \frac{x}{5}$ and $Sx = \frac{3x}{5}$, for all x in X. Consider the sequence $\{x_n\} = \{1/n\}$. Clearly $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = 0$. Then S and T satisfy the property (S-B).

Example 2.4: Let X = [2, + ∞). Define S, T : X → X by $Tx = x + 1/2$ and $Sx = 2x + 1/2$, $\forall x \in X$.

Suppose property (S-B) holds; then there exists in X a sequence $\{x_n\}$ satisfying

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \text{ for some } z \in X.$$

Therefore

$$\lim_{n\to\infty} x_n = z - 1/2 \quad \text{and} \quad \lim_{n\to\infty} x_n = (2z - 1)/4.$$

Then $z = 1/2$, which is a contradiction since $1/2 \notin X$. Hence S and T do not satisfy the property (S-B).

3 Main Results

Theorem 3.1 .Let (X, M, N, *, ◊) is an intuitionistic fuzzy2 metric space with t- norm $t * t \geq t$ and t- co norm $t \lozenge t \leq t$ for some $t \in [0, 1]$ and the condition (IF2M-3) and (IF2M-8). Let A, B and S be self mappings of X into itself such that

(3.1) $AX \subset SX$ and $BX \subset SX$,

(3.2) (A, S) or (B, S) satisfies the property (S-B),

(3.3) there exists a number $k \in (0, 1)$ such that
$M(Ax, By,a, kt) > M(Ax, Sx,a, t) * M(Sx, By,a, t)$
and $N(Ax, By,a, kt) < N(Ax, Sx,a, t) * N(Sx, By,a, t)$ for all $x, y,a \in X$ and $Ax \neq By$

(3.4) (A, S) and (B, S) are weakly compatible,

(3.5) one of AX, BX or SX is a closed subset of X.

Then A, B and S have a unique common fixed point in X.

Proof . Suppose that (B, S) satisfies the property (S-B). Then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Sx_n = z \text{ for some } z \in X.$$

Since $BX \subset SX$, there exists in X a sequence $\{y_n\}$ such that $Bx_n = Sy_n$. Hence $\lim_{n\to\infty} Sy_n = z$.

Let us show that $\lim_{n\to\infty} Ay_n = z$. Indeed, in view of (3.3), we have

$$\begin{aligned} M(Ay_n, Bx_n,a, kt) &> M(Ay_n, Sy_n,a, t) * M(Sy_n, Bx_n,a, t) \\ &> M(Ay_n, Bx_n,a, t) * M(By_n, Bx_n,a, t) \\ &> M(Ay_n, Bx_n,a, t) * 1 \end{aligned}$$

$$M(Ay_n, Bx_n,a, kt) > M(Ay_n, Bx_n,a, t)$$

And

$$N(Ay_n, Bx_n, a, kt) < N(Ay_n, Sy_n, a, t) \Diamond N(Sy_n, Bx_n, a, t)$$
$$< N(Ay_n, Bx_n, a, t) \Diamond N(By_n, Bx_n, a, t)$$
$$< N(Ay_n, Bx_n, a, t) \Diamond 0$$
$$N(Ay_n, Bx_n, a, kt) < N(Ay_n, Bx_n, a, t)$$

Therefore by Lemma 2.2, we deduce that $\lim_{n\to\infty} Ay_n = z$.

Suppose SX is a closed subset of X. Then z = Su for some $u \in X$. Subsequently, we have

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Sx_n = Su$$

By (3.3), we have

$$M(Au, Bx_n, a, kt) > M(Au, Su, a, t) * M(Su, Bx_n, a, t)$$

And
$$N(Au, Bx_n, a, kt) < N(Au, Su, a, t) \Diamond N(Su, Bx_n, a, t)$$

Letting $n\to\infty$, we obtain

$$M(Au, Su, a, kt) > M(Au, Su, a, t) * M(Su, Su, a, t)$$
$$> M(Au, Su, a, t) * 1$$
$$M(Au, Su, a, kt) > M(Au, Su, a, t)$$

And
$$N(Au, Su, a, kt) < N(Au, Su, a, t) \Diamond N(Su, Su, a, t)$$
$$< N(Au, Su, a, t) \Diamond 0$$
$$N(Au, Su, a, kt) < N(Au, Su, a, t)$$

Therefore by Lemma 2.2**,** we have Au = Su.

The weak compatibility of A and S implies that ASu = SAu and then AAu = ASu = SAu = SSu.

On the other hand, since $AX \subset SX$, there exists a point $v \in X$ such that Au = Sv. We claim that Sv = Bv.

Using (3.3), we have

$$M(Au, Bv, a, kt) > M(Au, Su, a, t) * M(Su, Bv, a, t)$$
$$> M(Au, Au, a, t) * M(Au, Bv, a, t)$$
$$> 1 * M(Au, Bv, a, t)$$
$$M(Au, Bv, a, kt) > M(Au, Bv, a, t)$$

And
$$N(Au, Bv, a, kt) < N(Au, Su, a, t) \Diamond N(Su, Bv, a, t)$$
$$< N(Au, Au, a, t) \Diamond N(Au, Bv, a, t)$$
$$< 0 \Diamond N(Au, Bv, a, t)$$
$$N(Au, Bv, a, kt) < N(Au, Bv, a, t)$$

Therefore by Lemma 2.2, we have Au = Bv.

Thus Au = Su = Sv = Bv. The weak compatibility of B and S implies

BSv = SBv and then BBv = BSv = SBv =SSv.

Let us show that Au is a common fixed point of A, B and S. In view of (3.3), it follows that

$$M(AAu, Bv, a, kt) > M(AAu, SAu, a, t) * M(SAu, Bv, a, t)$$
$$> M(AAu, AAu, a, t) * M(AAu, Au, a, t)$$
$$> 1 * M(AAu, Au, a, t)$$
$$M(AAu, Au, a, kt) > M(AAu, Au, a, t).$$

And
$$N(AAu, Bv, a, kt) < N(AAu, SAu, a, t) \Diamond N(SAu, Bv, a, t)$$
$$< N(AAu, AAu, a, t) \Diamond N(AAu, Au, a, t)$$
$$< 0 \Diamond N(AAu, Au, a, t)$$
$$N(AAu, Au, a, kt) < N(AAu, Au, a, t).$$

Therefore by Lemma 2.2**,** we have AAu = Au = SAu and Au is a common fixed point of A and S. Similarly, we prove that Bv is a common fixed point of B and S. Since Au = Bv, we conclude that Au is a common fixed point of A, B and S.

If Au = Bu = Su = u and Av = Bv = Sv = v, then by (3.3), we have

$$M(Au, Bv, a, kt) > M(Au, Su, a, t) * M(Su, Bv, a, t)$$
$$M(u, v, a, kt) > M(u, u, a, t) * M(u, v, a, t)$$
$$M(u, v, a, kt) > 1 * M(u, v, a, t)$$
$$M(u, v, a, kt) > M(u, v, a, t).$$

and

$$N(Au, Bv,a, kt) < N(Au, Su,a, t) \Diamond N(Su, Bv,a, t)$$
$$N(u, v,a, kt) < N(u, u,a, t) \Diamond N(u, v,a, t)$$
$$N(u, v,a, kt) < 0 \Diamond N(u, v,a, t)$$
$$N(u, v,a, kt) < N(u, v,a, t).$$

By Lemma 2.2, we have u = v and the common fixed point is unique. This completes the proof of the theorem.

Theorem 3.2 : Let $(X, M, N, *, \Diamond)$ is an intuitionistic fuzzy2 metric space with t- norm $t * t \geq t$ and t- co norm $t \Diamond t \leq t$ for some $t \in [0, 1]$ and the condition (IF2M-3) and (IF2M-8). Let A, B, S and T be self-mappings of X into itself such that

(3.6) $AX \subset TX$ and $BX \subset SX$,

(3.7) (A, S) or (B, T) satisfies the property (S-B),

(3.8) there exists a number $k \in (0, 1)$, such that

$$[1 + pM (Sx, Ty,a, kt)] * M (Ax, By,a, kt) \geq p [M (Ax, Sx,a, kt) * M (By, Ty,a, kt) + M (Ax, Ty,a, kt) * M (By, Sx,a, kt)] + M (Sx, Ty,a, t) * M (Ax, Sx,a, t) * M (By, Ty,a, t) * M (By, Sx,a, t) * M (Ax, Ty,a, (2 - \alpha) t)$$

$$[1 + pN (Sx, Ty,a, kt)] \Diamond N (Ax, By,a, kt) \leq p [N (Ax, Sx,a, kt) \Diamond N (By, Ty,a, kt) + N (Ax, Ty,a, kt) \Diamond N (By, Sx,a, kt)] + N (Sx, Ty,a, t) \Diamond N (Ax, Sx,a, t) \Diamond N (By, Ty,a, t) \Diamond N (By, Sx,a, t) \Diamond N (Ax, Ty,a, (2 - \alpha) t)$$

for all $x, y,a \in X$, $p \geq 0$ and $\alpha \in (0, 2)$.

(3.9) The pairs (A, S) and (B, T) are weakly compatible,

(3.10) One of AX, BX, SX or TX is a closed subset of X.

Then A, B, S and T have a unique common fixed point in X.

Proof . Suppose that (B, T) satisfies the property (S-B). Then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

Since $BX \subset SX$, there exists in X a sequence $\{y_n\}$ such that $Bx_n = Sy_n$. Hence $\lim_{n\to\infty} Sy_n = z$. Let us show that $\lim_{n\to\infty} Ay_n = z$. Indeed, in view of (3.8) for $\alpha = 1 - q$, $q \in (0, 1)$, we have

$$[1 + pM(Sy_n, Tx_n,a, kt)] * M(Ay_n, Bx_n,a, kt) \geq p [M(Ay_n, Sy_n,a, kt) * M(Bx_n, Tx_n,a, kt) + M(Ay_n, Tx_n,a,kt) * M(Bx_n, Sy_n,a, kt)] + M(Sy_n, Tx_n,a, t) * M(Ay_n, Sy_n,a, t) * M(Bx_n, Tx_n,a, t) * M(Bx_n, Sy_n,a, t) * M(Ay_n, Tx_n,a, (2 - \alpha)t)$$

$$M(Ay_n, Bx_n,a, kt) + p [M(Sy_n, Tx_n,a, kt) * M(Ay_n, Bx_n,a, kt)] \geq p [M(Ay_n, Sy_n,a, kt) * M(Bx_n, Tx_n,a, kt) + M(Ay_n, Tx_n,a, kt) * M(Bx_n, Sy_n,a, kt)] + M(Sy_n, Tx_n,a, t) * M(Ay_n, Sy_n,a, t) * M(Bx_n, Tx_n,a, t) * M(Bx_n, Sy_n,a, t) * M(Ay_n, Tx_n,a, (1 + q)t)$$

$$M(Ay_n, Bx_n,a, kt) + p [M(Bx_n, Tx_n,a, kt) * M(Ay_n, Bx_n,a, kt)] \geq p [M(Ay_n, Bx_n,a, kt) * M(Bx_n, Tx_n,a, kt) + M(Ay_n, Tx_n,a, kt) * M(Bx_n, Bx_n,a, kt)] + M(Bx_n, Tx_n,a, t) * M(Ay_n, Bx_n,a, t) * M(Bx_n, Tx_n,a, t) * M(Bx_n, Bx_n,a, t) *M(Ay_n, Tx_n,Bx_n, t) *M(Ay_n, Bx_n, a, qt/2) * M(Bx_n, Tx_n,a, qt/2)$$

And

$$[1 + pN(Sy_n, Tx_n,a, kt)] \Diamond N(Ay_n, Bx_n,a, kt) \leq p [N(Ay_n, Sy_n,a, kt) \Diamond N(Bx_n, Tx_n,a, kt) + N(Ay_n, Tx_n,a,kt) \Diamond N(Bx_n, Sy_n,a, kt)] + N(Sy_n, Tx_n,a, t) \Diamond N(Ay_n, Sy_n,a, t) \Diamond N(Bx_n, Tx_n,a, t) \Diamond N(Bx_n, Sy_n,a, t) \Diamond N(Ay_n, Tx_n,a, (2 - \alpha)t)$$

$$N(Ay_n, Bx_n,a, kt) + p [N(Sy_n, Tx_n,a, kt) \Diamond N(Ay_n, Bx_n,a, kt)]$$

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$$\leq p\,[N(Ay_n, Sy_n,a, kt) \lozenge N(Bx_n, Tx_n,a, kt) + N(Ay_n, Tx_n,a, kt) \lozenge N(Bx_n, Sy_n,a, kt)] + N(Sy_n, Tx_n,a, t) \lozenge N(Ay_n, Sy_n,a, t) \lozenge N(Bx_n, Tx_n,a, t) \lozenge N(Bx_n, Sy_n,a, t) \lozenge N(Ay_n, Tx_n,a, (1 + q)t)$$

$$N(Ay_n, Bx_n,a, kt) + p\,[N(Bx_n, Tx_n,a, kt) \lozenge N(Ay_n, Bx_n,a, kt)]$$
$$\leq p\,[N(Ay_n, Bx_n,a, kt) \lozenge N(Bx_n, Tx_n,a, kt) + N(Ay_n, Tx_n,a, kt) \lozenge N(Bx_n, Bx_n,a, kt)] + N(Bx_n, Tx_n,a, t) \lozenge N(Ay_n, Bx_n,a, t) \lozenge N(Bx_n, Tx_n,a, t) \lozenge N(Bx_n, Bx_n,a, t) \lozenge N(Ay_n, Tx_n,Bx_n, t) \lozenge N(Ay_n, Bx_n, a, qt/2) \lozenge N(Bx_n, Tx_n,a, qt/2)$$

Thus it follows that

$M(Ay_n, Bx_n,a, kt) \geq M(Bx_n,Tx_n,a, t)*M(Ay_n,Bx_n,a,qt/2)*M(Bx_n,Tx_n,a,qt/2)$

And $N(Ay_n, Bx_n,a, kt) \leq N(Bx_n,Tx_n,a, t)\lozenge N(Ay_n,Bx_n,a,qt/2)\lozenge N(Bx_n,Tx_n,a,qt/2)$

Since the t-norm * and t-conorm $\lozenge$ are continuous and M , N are also is continuous, letting $q \to 1$, we have

$M(Ay_n, Bx_n,a, kt) \geq M(Bx_n, Tx_n,a, t) * M(Ay_n, Bx_n,a, t/2)$

And $N(Ay_n, Bx_n,a, kt) \leq N(Bx_n, Tx_n,a, t) \lozenge N(Ay_n, Bx_n,a, t/2)$

It follows that

$\lim_{n\to\infty} M(Ay_n, Bx_n,a, kt) \geq \lim_{n\to\infty} M(Ay_n, Bx_n,a, t)$

and $\lim_{n\to\infty} N(Ay_n, Bx_n,a, kt) \leq \lim_{n\to\infty} N(Ay_n, Bx_n,a, t)$

and we deduce that $\lim_{n\to\infty} Ay_n = z$.

Suppose SX is a closed subset of X. Then z = Su for some $u \in X$. Subsequently, we have

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sy_n = Su.$$

By (3.8) with $\alpha = 1$, we have

$$[1 + pM(Su, Tx_n,a, kt)] * M(Au, Bx_n,a, kt)$$
$$\geq p\,[M(Au, Su,a, kt) * M(Bx_n, Tx_n,a, kt) + M(Au, Tx_n,a,kt) * M(Bx_n, Su,a, kt)] + M(Su, Tx_n,a, t) * M(Au, Su,a, t) * M(Bx_n, Tx_n,a, t) * M(Bx_n, Su,a, t) * M(Au, Tx_n,a, t)$$

$$M(Au, Bx_n,a, kt) + p[M(Su, Tx_n,a, kt)] * M(Au, Bx_n,a, kt)]$$
$$\geq p[M(Au, Su,a,kt) * M(Bx_n, Tx_n,a,kt) + M(Au, Tx_n,a,kt) * M(Bx_n, Su,a, kt)] + M(Su, Tx_n,a, t) * M(Au, Su,a, t) * M(Bx_n, Tx_n,a, t) * M(Bx_n, Su,a, t) * M(Au, Tx_n,a, t)$$

Taking the $\lim_{n\to\infty}$, we have

$$M(Au, Su,a, kt) \geq p[(Au, Su,a, kt) * M(Su, Su,a,kt)] + M(Su, Su,a,t) * M (Au, Su,a, t) * M (Su, Su,a, t) * M (Su, Su,a, t) * M (Au, Su,a, t)$$

And

$$[1 + pN(Su, Tx_n,a, kt)] \lozenge N(Au, Bx_n,a, kt)$$
$$\leq p\,[N(Au, Su,a, kt) \lozenge N(Bx_n, Tx_n,a, kt) + N(Au, Tx_n,a,kt) \lozenge N(Bx_n, Su,a, kt)] + N(Su, Tx_n,a, t) \lozenge N(Au, Su,a, t) \lozenge N(Bx_n, Tx_n,a, t) \lozenge N(Bx_n, Su,a, t) \lozenge N(Au, Tx_n,a, t)$$

$$N(Au, Bx_n,a, kt) + p[N(Su, Tx_n,a, kt)] \lozenge N(Au, Bx_n,a, kt)]$$
$$\leq p[N(Au, Su,a,kt) \lozenge N(Bx_n, Tx_n,a,kt) + N(Au, Tx_n,a,kt) \lozenge N(Bx_n, Su,a, kt)] + N(Su, Tx_n,a, t) \lozenge N(Au, Su,a, t) \lozenge N(Bx_n, Tx_n,a, t) \lozenge N(Bx_n, Su,a, t) \lozenge N(Au, Tx_n,a, t)$$

Taking the $\lim_{n\to\infty}$, we have

$$N(Au, Su,a, kt) \leq p[N(Au, Su,a, kt) \lozenge N(Su, Su,a,kt)] + N(Su, Su,a,t)\lozenge N (Au, Su,a, t) \lozenge N (Su, Su,a, t) \lozenge N (Su, Su,a, t) \lozenge N (Au, Su,a, t)$$

These gives

$M (Au, Su,a, kt) \geq M (Au, Su,a, t)$ and $N (Au, Su,a, kt) \leq N (Au, Su,a, t)$

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Therefore by Lemma 2.2, we have Au = Su. The weak compatibility of A and S implies that ASu = SAu and then AAu = ASu = SAu = SSu. On the other hand, since $AX \subset TX$, there exists a point $v \in X$ such that Au = Tv. We claim that Tv = Bv using (3.8) with $\alpha = 1$, we have

$$[1 + pM(Su, Tv,a, kt)] * M(Au, Bv,a, kt) \geq p[M(Au, Su,a, kt) * M(Bv, Tv,a, kt) + M(Au, Tv,a, kt) * M(Bv, Su,a, kt)] + M(Su, Tv,a, t) * M(Au, Su,a, t) * M(Bv, Tv,a, t) * M(Bv, Su,a, t) * M(Au, Tv,a, t)$$

$$M(Au, Bv,a, kt) + p[M(Su, Tv,a, kt) * M(Au, Bv,a, kt)] \geq p[M(Au, Su,a, kt) * M(Bv, Tv,a, kt) + M(Au, Tv,a, kt) * M (Bv, Su,a, kt)] + M (Su, Tv,a, t) * M (Au, Su,a, t) * M (Bv, Tv,a, t) * M (Bv, Su,a, t) * M (Au, Tv,a, t)$$

And

$$[1 + pN(Su, Tv,a, kt)] \Diamond N(Au, Bv,a, kt) \leq p[N(Au, Su,a, kt) \Diamond N(Bv, Tv,a, kt) + N(Au, Tv,a, kt) \Diamond N(Bv, Su,a, kt)] + N(Su, Tv,a, t) \Diamond N(Au, Su,a, t) \Diamond N(Bv, Tv,a, t) \Diamond N(Bv, Su,a, t) \Diamond N(Au, Tv,a, t)$$

$$N(Au, Bv,a, kt) + p[N(Su, Tv,a, kt) \Diamond N(Au, Bv,a, kt)] \leq p[N(Au, Su,a, kt) \Diamond N(Bv, Tv,a, kt) + N(Au, Tv,a, kt) \Diamond N (Bv, Su,a, kt)] + N (Su, Tv,a, t) \Diamond N (Au, Su,a, t) \Diamond N (Bv, Tv,a, t) \Diamond N (Bv, Su,a, t) \Diamond N (Au, Tv,a, t)$$

Thus it follows that

$M(Au, Bv,a, kt) \geq M(Au, Bv,a, t)$ and $N(Au, Bv,a, kt) \leq N(Au, Bv,a, t)$

Therefore by Lemma 2.2, we have Au = Bv.

Thus Au = Su = Tv = Bv. The weak compatibility of B and T implies that BTv = TBv and TTv = TBv = BTv = BBv. Let us show that Au is a common fixed point of A, B, S and T. In view of (3.8) with $\alpha = 1$, we have

$$[1 + pM(SAu, Tv,a, kt)] * M(AAu, Bv,a, kt) \geq p[M(AAu, SAu,a, kt) * M(Bv, Tv,a, kt) + M(AAu, Tv,a, kt) * M(Bv, SAu,a, kt)] + M(SAu, Tv,a, t) * M(AAu, SAu,a, t) * M(Bv, Tv,a, t) * M(Bv, SAu,a, t) * M(AAu, Tv,a, t)$$

$$M(AAu, Bv,a, kt) + p[M(SAu, Tv,a, kt) * M(AAu, Bv,a, kt)] \geq p[M(AAu, SAu,a, kt) * M(Bv, Tv,a, kt) + M (AAu, Tv,a, kt) * M(Bv, SAu,a, kt)] + M(SAu, Tv,a, t) * M(AAu, SAu,a, t) * M(Bv, Tv,a, t) * M(Bv, SAu,a, t) * M(AAu, Tv,a, t)$$

$$M(AAu, Au,a, kt) + p[M(AAu, Au,a, kt) * M(AAu, Au,a, kt)] \geq p[M(AAu, AAu,a, kt) * M(Au, Au,a, kt) + M(AAu, Au,a, kt) * M(Au, AAu,a, kt)] + M(AAu, Au,a, t) * M(AAu, AAu,a, t) * M(Au, Au,a, t) * M(Au, AAu,a, t) * M(AAu, Au,a, t)$$

$$[1 + pN(SAu, Tv,a, kt)] \Diamond N(AAu, Bv,a, kt) \leq p[N(AAu, SAu,a, kt) \Diamond N(Bv, Tv,a, kt) + N(AAu, Tv,a, kt) \Diamond N(Bv, SAu,a, kt)] + N(SAu, Tv,a, t) \Diamond N(AAu, SAu,a, t) \Diamond N(Bv, Tv,a, t) \Diamond N(Bv, SAu,a, t) \Diamond N(AAu, Tv,a, t)$$

$$N(AAu, Bv,a, kt) + p[N(SAu, Tv,a, kt) \Diamond N(AAu, Bv,a, kt)] \leq p[N(AAu, SAu,a, kt) \Diamond N(Bv, Tv,a, kt) + N (AAu, Tv,a, kt) \Diamond N(Bv, SAu,a, kt)] + N(SAu, Tv,a, t) \Diamond N(AAu, SAu,a, t) \Diamond N(Bv, Tv,a, t) \Diamond N(Bv, SAu,a, t) \Diamond N(AAu, Tv,a, t)$$

$$N(AAu, Au,a, kt) + p[N(AAu, Au,a, kt) \Diamond N(AAu, Au,a, kt)] \leq p[N(AAu, AAu,a, kt) \Diamond N(Au, Au,a, kt) + N(AAu, Au,a, kt) \Diamond N(Au, AAu,a, kt)] + N(AAu, Au,a, t) \Diamond N(AAu, AAu,a, t)$$

$\Diamond$ N(Au, Au,a, t) $\Diamond$ N(Au, AAu,a, t) $\Diamond$ N(AAu, Au,a, t)

Thus it follows that

M(AAu, Au,a kt) $\geq$ M(AAu, Au,a, t) and N(AAu, Au,a kt) $\leq$ N(AAu, Au,a, t)

Therefore by Lemma 2.2, we have Au = AAu = SAu and Au is a common fixed point of A and S.

Similarly, we prove that Bv is a common fixed point of B and T. Since Au = Bv, we conclude that Au is a common fixed point of A, B, S and T.

If Au = Bu = Su = Tu = u and Av = Bv = Sv = Tv = v, then by (3.8) with α = 1, we have

[1 + pM(Su, Tv,a, kt)] * M(Au, Bv,a, kt)
$\geq$ p[M(Au, Su,a, kt) * M(Bv, Tv,a, kt) + M(Au, Tv,a, kt)
* M(Bv, Su,a, kt)] + M(Su, Tv,a, t) * M(Au, Su,a, t)
* M(Bv, Tv,a, t) * M(Bv, Su,a, t) * M(Au, Tv,a, t)

M(u, v,a, kt) + p[M(u, v,a, kt) * M(u, v,a, kt)]
$\geq$ p[M(u, u,a, kt) * M(v, v,a, kt) + M(u, v,a, kt)
* M(v, u,a, kt)] + M(u, v,a,t) * M(u, u,a, t)
* M(v, v,a, t) * M(v, u,a, t) * M(u, v,a, t)

[1 + pN(Su, Tv,a, kt)] $\Diamond$ N(Au, Bv,a, kt)
$\geq$ p[N(Au, Su,a, kt) $\Diamond$ N(Bv, Tv,a, kt) + N(Au, Tv,a, kt)
$\Diamond$ N(Bv, Su,a, kt)] + N(Su, Tv,a, t) $\Diamond$ N(Au, Su,a, t)
$\Diamond$ N(Bv, Tv,a, t) $\Diamond$ N(Bv, Su,a, t) $\Diamond$ N(Au, Tv,a, t)

N(u, v,a, kt) + p[N(u, v,a, kt) $\Diamond$ N(u, v,a, kt)]
$\geq$ p[N(u, u,a, kt) $\Diamond$ N(v, v,a, kt) + N(u, v,a, kt)
$\Diamond$ N(v, u,a, kt)] + N(u, v,a,t) $\Diamond$ N(u, u,a, t)
$\Diamond$ N(v, v,a, t) $\Diamond$ N(v, u,a, t) $\Diamond$ N(u, v,a, t)

This gives

M(u, v,a, kt) $\geq$ M(u, v,a, t) and N(u, v,a, kt) $\leq$ N(u, v,a, t)

By Lemma 2.2, we have u = v and the common fixed point is a unique. This completes the proof of the theorem.

If we put p = 0, we get the following result:

Corollary 3.1 . Let (X, M, N, *, $\Diamond$) is an intuitionistic fuzzy2 metric space with t- norm $t * t \geq t$ and t- co norm $t \Diamond t \leq t$ for some $t \in [0, 1]$ and the condition (IF2M-3) and (IF2M-8). Let A, B, S and T be self-mappings of X into itself such that

(3.11) AX $\subset$ TX and BX $\subset$ SX,

(3.12) (A, S) or (B, T) satisfies the property (S-B),

(3.13) there exists a number $k \in (0, 1)$, such that

M(Ax, By,a, kt) $\geq$ M(Sx, Ty,a, t) * M(Ax, Sx,a, t) * M(By, Ty,a, t)
* M(By, Sx,a, t) * M(Ax, Ty,a, (2 - α) t)

N(Ax, By,a, kt) $\leq$ N(Sx, Ty,a, t) $\Diamond$ N(Ax, Sx,a, t) $\Diamond$ N(By, Ty,a, t)
$\Diamond$ N(By, Sx,a, t) $\Diamond$ N(Ax, Ty,a, (2 - α) t)

for all x, y,a $\in$ X and $\alpha \in (0, 2)$.

(3.14) (A, S) and (B, T) are weakly compatible,

(3.15) one of AX, BX, SX or TX is a closed subset of X.

Then A, B, S and T have a unique common fixed point in X.

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